

# Generation of Neutrino Masses and Mixings in Gauge Theories

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I review models which present large flavor mixings of the lepton sector based on the gauge theory. (Invited talk at WIN99)

## 1 Introduction

Recent experimental data of neutrinos make big impact on the neutrino masses and their mixings. Most exciting one is the results at Super-Kamiokande on the atmospheric neutrinos, which indicate the large neutrino flavor oscillation of  $\nu_\mu \rightarrow \nu_\tau$ <sup>1</sup>. Solar neutrino data also provide the evidence of the neutrino oscillation, however this problem still uncertain<sup>2</sup>.

What can we learn from these results? We want to get clues for origins of neutrino masses and neutrino flavor mixings. In particular, we want to understand why the neutrino mixing is large compared with the quark sector. Now we should discuss these problems in connection with the quark sector.

## 2 Phenomenological Aspect of Neutrino Masses and Mixings

Our starting point as to the neutrino mixing is the large  $\nu_\mu \rightarrow \nu_\tau$  oscillation of the atmospheric neutrino oscillation with  $\Delta m_{\text{atm}}^2 = (1 \sim 6) \times 10^{-3} \text{eV}^2$  and  $\sin^2 2\theta_{\text{atm}} \geq 0.9$  which are derived from the recent data of the atmospheric neutrino deficit at Super-Kamiokande<sup>1</sup>. In the solar neutrino problem<sup>2</sup>, there are three solutions: the MSW small angle solution, the MSW large angle solution and the vacuum solution. These mass difference scales are much smaller than the atmospheric one. Once we put  $\Delta m_{\text{atm}}^2 = \Delta m_{32}^2$  and  $\Delta m_\odot^2 = \Delta m_{21}^2$ , there are two typical mass patterns:  $m_3 \gg m_2 \gg m_1$  and  $m_3 \simeq m_2 \simeq m_1$ .

The neutrino mixing is defined as  $\nu_\alpha = U_{\alpha i} \nu_i$ , where  $\alpha$  denotes the flavor  $e, \mu, \tau$  and  $i$  denotes mass eigenvalues 1, 2, 3. Now we have two typical mixing patterns:

$$U_{\text{MNS}} = \begin{pmatrix} 1 & U_{e2} & U_{e3} \\ U_{\mu 1} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ U_{\tau 1} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & U_{e3} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (1)$$

the first one is the single maximal mixing pattern, in which the solar neutrino deficit is explained by the small angle MSW solution, and the other is the

bi-maximal mixings pattern, in which the solar neutrino deficit is explained by the just so solution. In both case  $U_{e3}$  is constrained by the CHOOZ Data. introduced. These quarks and leptons

### 3 Neutrino Masses and Mixings in the GUT

The left handed neutrino masses are supposed to be at most  $\mathcal{O}(1)\text{eV}$ . In the case of Majorana neutrino, we know two classes of models which lead naturally to a small neutrino mass: (i) models in which the seesaw mechanism works and (ii) those in which the neutrino mass is induced by a radiative correction. The central idea of models (i) supposes some higher symmetry which is broken at an high energy scale. If this symmetry breaking takes place so that it allows the right-handed neutrino to have a mass, and a small mass induced for the left handed neutrino by the seesaw mechanism. In the classes of model (ii) one introduces a scalar particle with a mass of the order of the electroweak (EW) energy scale which breaks the lepton number in the scalar sector. A left-handed neutrino mass is then induced by a radiative correction from a scalar loop. This model requires some new physics at the EW scale.

**SU(5) GUT:** In the standpoint of the quark-lepton unification, the charged lepton mass matrix is connected with the down quark one. The mixing following from the charged lepton mass matrix may be considered to be small like quarks in the hierarchical base. However, this expectation is not true if the mass matrix is non-Hermitian. In the SU(5), fermions belong **10** and **5\***:

$$\mathbf{10} : \chi_{ab} = u^c + Q + e^c, \quad \mathbf{5}^* : \psi^a = d^{c1} + L, \quad (2)$$

where  $Q$  and  $L$  are SU(2) doublets of quarks and leptons, respectively. The Yukawa couplings are given by  $10_i 10_j 5_H$  (up-quarks) and  $5_i^* 10_j 5_H^*$  (down-quarks and charge leptons) ( $i, j=1, 2, 3$ ). Therefore we get  $m_E = m_D^T$  at the GUT scale.

It should be noticed that observed quark mass spectra and the CKM matrix only constrains the down quark mass matrix typically as follows:

$$m_{\text{down}} \sim K_D \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^4 \\ x & \lambda^2 & \lambda^2 \\ y & z & 1 \end{pmatrix} \quad \text{with} \quad \lambda = 0.22. \quad (3)$$

Three unknown  $x$ ,  $y$ ,  $z$  are related to the left-handed charged lepton mixing due to  $m_E = m_D^T$ . The left(right)-handed down quark mixings are related to the right(left)-handed charged lepton mixings in the SU(5). Therefore, there is a source of the large flavor mixing in the charged lepton sector if  $z \simeq 1$  is derived from some models. This mechanism was nicely used by some authors<sup>3,4,5</sup>.

In the case of the SO(10) GUT, SO(10) breaking may lead to the large mixing in the charged lepton sector if an asymmetric interaction in the family space exists<sup>4</sup>. In conclusion, the  $\nu_\mu - \nu_\tau$  mixing could be maximal in some GUT models, which are consistent with the quark sector.

**See-saw enhancement:** The large mixing may come from the neutrino sector. It could be obtained in the see-saw mechanism as a consequence of a certain structure of the right-handed Majorana mass matrix<sup>6,7</sup>. That is the so called see-saw enhancement of the neutrino mixing due to the cooperation between the Dirac and Majorana mass matrices.

Mass matrix of light Majorana neutrinos  $m_\nu$  has the following form

$$m_\nu \simeq -m_D M_R^{-1} m_D^T, \quad (4)$$

where  $m_D$  is the neutrino Dirac mass matrix and  $M_R$  is the Majorana mass matrix of the right-handed neutrino components. Then, the lepton mixing matrix is<sup>6</sup>  $V_\ell = S_\ell^\dagger \cdot S_\nu \cdot V_s$ , where  $S_\ell$ ,  $S_\nu$  are transformations which diagonalize the Dirac mass matrices of charged leptons and neutrinos, respectively. The  $V_s$  specifies the effect of the see-saw mechanism, i.e. the effects of the right-handed Majorana mass matrix. It is determined by

$$V_s^T m_{ss} V_s = \text{diag}(m_1, m_2, m_3), \quad \text{with} \quad m_{ss} = -m_D^{diag} M_R^{-1} m_D^{diag}. \quad (5)$$

In the case of two generations, the mixing matrix  $V_s$  is easily investigated in terms of one angle  $\theta_s$ . This angle could be maximal under the some conditions of parameters in the Dirac mass matrix and right handed Majorana mass matrix. That is the enhancement due to the see-saw mechanism. The rich structure of right-handed Majorana mass matrix can lead to the maximal flavor mixing of neutrinos.

**Radiative neutrino mass:** In the class of models in (ii), neutrino masses are induced from the radiative corrections. The typical one is the Zee model, in which charged gauge singlet scalar induces the neutrino mass<sup>8</sup>. In this model, the previous predictions are consistent with LSND data and atmospheric neutrino data. Then the solar neutrino deficit was explained by introducing the sterile neutrino. However new solution has been found in the framework of the Zee model. In the case of the inverse hierarchy  $m_1 \simeq m_2 \gg m_3$ , the bi-maximal mixing, which is consistent with atmospheric and solar neutrinos, is obtained<sup>9</sup>.

The MSSM with R-parity violation can also give the neutrino masses and mixings. The MSSM allows renormalizable B and L violation. The R-parity conservation forbids the B and L violation in the superpotential in order to avoid the proton decay. However the proton decay is avoided in the tree level

if either of B or L violating term vanishes. The simplest model is the bi-linear R-parity violating model with  $\epsilon_i H_u L_i$  for the lepton-Higgs coupling<sup>10</sup>. This model provides the large mixing which is consistent with atmospheric and solar neutrinos.

#### 4 Flavor Symmetry and Large Mixings

In the previous discussions, we assumed the family structure in the mass matrices. However masses and mixings may suggest the some flavor symmetry. The simple flavor symmetry is U(1), which was discussed intensively by Ramond et al.<sup>11</sup>. In their model, they assumed (1) Fermions carry U(1) charge, (2) U(1) is spontaneously broken by  $\langle \theta \rangle$ , in which  $\theta$  is the EW singlet with U(1) charge -1, and (3) Yukawa couplings appear as effective operators

$$h_{ij}^D Q_i \bar{d}_j H_d \left( \frac{\theta}{\Lambda} \right)^{m_{ij}} + h_{ij}^U Q_i \bar{u}_j H_u \left( \frac{\theta}{\Lambda} \right)^{n_{ij}} + \dots, \quad (6)$$

where  $\langle \theta \rangle / \Lambda = \lambda \simeq 0.22$ . The powers  $m_{ij}$  and  $n_{ij}$  are determined from the U(1) charges of fermions in order that the effective operators are U(1) invariants. The U(1) charges of the fermions are fixed by the experimental data of the fermion masses and mixings. Then the model has anomalous U(1).

Another typical flavor symmetry is  $S_3$ . The  $S_{3L} \times S_{3R}$  symmetric mass matrix is so called the democratic mass matrix<sup>12</sup>, which needs the large rotation in order to move to the diagonal base. In the quark sector, this large rotation is canceled each other between down quarks and up quarks. However, the situation of the lepton sector is very different from the quark sector if the effective neutrino mass matrix  $m_{LL}^\nu$  is far from the democratic one and the charged lepton one is still the democratic one. Let us consider the neutrino mass matrices, which provide large mixings<sup>13</sup>: The typical one is

$$M_\nu = c_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & \epsilon_\nu & 0 \\ \epsilon_\nu & 0 & 0 \\ 0 & 0 & \delta_\nu \end{pmatrix}, \quad \text{or} \quad + \begin{pmatrix} -\epsilon_\nu & 0 & 0 \\ 0 & \epsilon_\nu & 0 \\ 0 & 0 & \delta_\nu \end{pmatrix}, \quad (7)$$

where the first term is the  $S_{3L}$  symmetric effective mass matrix and the second or the third is the  $S_{3L}$  breaking one. In the case of the first breaking matrix, the large mixing of (1–2) family sector is completely canceled between the neutrino and the charged lepton sectors, however the large mixing of the (2–3) family in the charged lepton sector is not canceled. So we have the large mixing in the lepton flavor mixing matrix. If we adopt the latter symmetry breaking matrix<sup>13,14</sup>, we obtain the lepton mixing matrix to be near bi-maximal because the large mixings from the charged lepton mass matrix cannot be canceled. This

case can accommodate the "just-so" scenario for the solar neutrino problem due to neutrino oscillation in vacuum. recent data

## 5 Summary

Models depends on three phenomenological aspects. Is the mixing pattern the single maximal mixing or bi-maximal mixing ? Is there sterile neutrino ? Are the neutrino masses degenerated or hirarchical ones? More precise solar neutrino data will answer the first and second questions. More precise atmospheric neutrino data and the long baseline experiments can answer the second question. The double beta dacay experiments may answer the last question. We need more data in order to establish the model as well as more theoretical studies.

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